Complexity of Life via Collective Mind

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Abstract

Collective mind is introduced as a set of simple intelligent units (say, neurons, or interacting agents), which can communicate by exchange of information without explicit global control. Incomplete information is compensated by a sequence of random guesses symmetrically distributed around expectations with prescribed variances. Both the expectations and variances are the invariants characterizing the whole class of agents. These invariants are stored as parameters of the collective mind, while they contribute into dynamical formalism of the agents' evolution, and in particular, into the reflective chains of their nested abstract images of the selves and non-selves. The proposed model consists of the system of stochastic differential equations in the Langevin form representing the motor dynamics, and the corresponding Fokker-Planck equation representing the mental dynamics (Motor dynamics describes the motion in physical space, while mental dynamics simulates the evolution of initial errors in terms of the probability density). The main departure of this model from Newtonian and statistical physics is due to a feedback from the mental to the motor dynamics which makes the Fokker-Planck equation nonlinear. Interpretation of this model from mathematical and physical viewpoints, as well as possible interpretation from biological, psychological, and social viewpoints are discussed. The model is illustrated by the dynamics of a dialog.

1. Introduction

The concept of collective mind has appeared recently as a subject of intensive scientific discussions from economical, social, ecological, and computational viewpoints (Huberman, B., 1988, Zak, M., 1993). It can be introduced as a set of simple units of intelligence (say, neurons, or interacting agents), which can communicate by exchange of information without an explicit global control. The objectives of the agents may be partly compatible and partly contradictory (i.e., they can cooperate or compete). The exchanging information may be at times inconsistent, often imperfect, non-deterministic and delayed. Nevertheless, observations of working insect colonies, social systems, and scientific communities suggest that such collectives of agents appear to be very successful in achieving global objectives, as well as in learning, memorizing, generalizing and predicting, due to their flexibility, adaptability to environmental changes, and creativity.

The objective of this paper is to introduce a dynamical formalism describing the evolution of the behavior of communicating agents. All the previous attempts to develop models for so called active systems (i.e., systems that possess certain degree of autonomy from the environment that allows them to perform motions that are not directly controlled from outside) have been based upon the principles of Newtonian and statistical mechanics (A. S. Mikhailov, 1990). These models appear to be so general that they predict not only physical, but also some biological and economical, as well as social patterns of behavior exploiting such fundamental properties of nonlinear dynamics as attractors. Not withstanding indisputable successes of that approach (neural networks, distributed active systems, etc.) there is still a fundamental limitation that characterizes these models on a dynamical level of description, they propose no difference between a solar system, a swarm of insects, and a stock market. Such a phenomenological reductionism is incompatible with the first principle of progressive biological evolution (I. Prigogine, 1980, H. Haken, 1988). According to this principle, the evolution of living systems is directed toward the highest levels of complexity if the complexity is measured by an irreducible number of different parts which interact in a well-regulated fashion (although in some particular cases deviations from this general tendency are possible). At the same time, the solutions to the models based upon dissipative Newtonian dynamics eventually approach attractors where the evolution stops (until a "master" reprograms the model). Therefore, such models fail to provide an autonomous progressive evolution of living systems (i.e. evolution leading to increase of complexity).

Let us now turn to the stochastic extension of Newtonian models. Actually, it is a well-established fact that evolution of life has a diffusion-based stochastic nature as a result of the multi-choice character of behavior of living systems. That means that the simplest living species must obey the second law of thermodynamics as physical particles do. However, then the evolution of living systems (during periods of their isolation) will be regressive since their entropy will increase (I. Prigogine, 1955). As pointed out by R. Gordon (1999), a stochastic motion describing physical systems does not have a sense of direction, and therefore, it cannot describe a progressive evolution. As an escape from this paradox, Gordon proposed a concept of differentiating waves (represented by traveling waves of chemical concentration or mechanical deformation) which are asymmetric by their nature, and this asymmetry creates a sense of direction toward progressive evolution. Although the concept of differentiating waves itself seems convincing, it raises several questions to be answered: Who or what arranges the asymmetry of the differentiating waves in the "right" direction? How to incorporate their formalism into statistical mechanics providing progressive evolution without a violation of the second law of thermodynamics? Thus, although the stochastic extension of Newtonian models can be arranged in many different ways (for instance, via relaxation of the Lipcshitz conditions, (M. Zak, 1992), or by means of opening escape-routes from the attractors), the progressive evolution of living systems cannot be provided.

The limitations discussed above have been addressed in several publications in which the authors were seeking a "border line" between living and non-living systems. It is worth noticing that one of the "most obvious" distinctive properties of the living systems, namely, their intentionality, can be formally disqualified by simple counter-examples; indeed, any mechanical (non-living) system has an "objective" to minimize action (the Hamilton principle) as well as any isolated diffusion-based stochastic (non-living) system has an "objective" to maximize the entropy production ("The Jaynes Principle," H. Haken, 1988). The departure from Newtonian models via introduction of dynamics with expectations and feedback from future has been proposed by B. Huberman and his associates (B. Huberman, 1993). Further departure which includes learning nested models of multi-agent systems were introduced by J. Vidal (J. Vidal, 1998). However, despite the fact that the non-Newtonian nature of living systems in these works was captured correctly, there is no global analytical model which would unify the evolution of the agent's state variables and their probabilistic characteristics such as expectations, self-images etc.

The objective of this paper is to develop a new mathematical formalism within the framework of classical dynamics that would allow one to capture the specific properties of natural or artificial living systems such as formation of the collective mind based upon abstract images of the selves and non-selves, exploitation of this collective mind for communications and predictions of future expected characteristics of evolution, as well as for making decisions and implementing the corresponding corrections if the expected scenario is different from the originally planned one. The approach is based upon our previous publications (M. Zak, 1999, 2000, and 2002) which postulate that even a primitive living species possesses additional non-Newtonian properties which are not included in the laws of Newtonian or statistical mechanics. These properties follow from a privileged ability of living systems to possess a self-image (a concept introduced in psychology) and to interact with it. The mathematical formalism is based upon coupling the classical dynamical system (with random components caused by uncertainties in initial conditions as well as by the Langevin forces) representing the motor dynamics with the corresponding Fokker-Planck equation describing the evolution of these uncertainties in terms of the probability density and representing the mental dynamics. The coupling is implemented by the information-based supervising forces that can be associated with the self-awareness. These forces fundamentally change the pattern of the probability evolution, and therefore, leading to a major departure of the behavior of living systems from the patterns of both Newtonian and statistical mechanics. Further extension, analysis, interpretation, and application of this approach to the collective-mind-based communicating agents will be addressed in this paper. It should be stressed that the proposed model is supposed to capture the signature of life on the phenomenological level, i.e., based only upon the observable behavior, and therefore, it will not include a bio-chemical machinery of metabolism. Such a limitation will not prevent one from using this model for developing artificial living systems as well as for studying some general properties of behavior of natural living systems. Although the proposed model is supposed to be applicable to both open and closed autonomous systems, the attention will be concentrated upon the latter since such properties of living systems as free will, prediction of future, decision making abilities, and especially, the phenomenology of mind, become more transparent there.

2. Reflective chains: what do you think I think you think...

We will start with the simplest model of two interacting agents assuming that each agent is represented by an inertionless classical point evolving in physical space. We will also assume that the next future position of each agent depends only upon its own present position and the present position of his opponent. Then their evolutionary model can be represented by the following system of differential equations:

$$\dot{x}_1 = f_1(x_1, x_2), \tag{1}$$

$$\dot{x}_2 = f_2(x_1, x_2) \tag{2}$$

Here x_1 and x_2 are the state variables for the agent 1 and the agent 2, respectively.

We will start with the assumption that these agents belong to the same class, and therefore, they know the structure of the whole system (1), (2). However, each of the agents may not know the initial condition of the other one, and therefore, he cannot calculate the current value of his opponent's state variable. As a result of that, the agents try to reconstruct these values using the images of their opponents. This process can be associated with the concept of reflection; in psychology reflection is defined as the ability of a person to create a self-nonself images and interact with them.

Let as turn first to the agent 1. In this view the system (1), (2) looks as following

$$\dot{x}_{11} = f_1(x_{11}, x_{21}), \tag{3}$$

$$\dot{x}_{21} = f_2(x_{21}, x_{121}) \tag{4}$$

where x_{11} is the self-image of the agent 1, $x_{21}$is the agent's 1 image of the agent 2, and x_{121} is the agent's 1 image of the agent's 2 image of the agent 1.

This system is not closed since it includes an additional 3-index variable x_{121}. In order to find the corresponding equation for this variable, one has to rewrite equations (3), (4) in the 3-index form. But it is easily verifiable that such form will include 4-index variables, etc., i.e., this chain of equations will never be closed. By interchanging the indices 1 and 2 in equations (3) and (4), one arrives at the system describing the view of the agent 2. The situation can be generalized from two- to n – dimensional systems. It is easy to calculate that the total number of equations for the m-th level of reflection, i.e., for the m-index variables, is

$$N_m = n^m. \tag{5}$$

Thus, the number of equations grows exponentially with the number of the levels of reflections, and it grows linearly with the dimensionality n of the original system. It should be noticed that for each m-th level of reflection, the corresponding system of equations always includes (m+1)-index variables, and therefore, it is always open. Hence, for any quantitative results, this system must be supplemented by a closure, i.e., by additional equations with respect to extra-variables. In order to illustrate how it can be done, let us first reduce equations (1) and (2) to the linear form

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2, \tag{6}$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2, \tag{7}$$

Taking the position of the agent 1, we can rewrite equation (6) in the form:

$$\dot{x}_1 = a_{11}x_i + a_{12}x_{21} \tag{8}$$

In which the unknown agent's 2 state variable x_2 is replaced with its value x_{21} to be predicted by the agent 1. Recalling that the agents 1 and 2 belong to the same class, it is reasonable to assume that the agent 1 knows the expected initial value χ_2^0 as well as the initial variance σ_2^0 of the agent's 2 state variable x_2. Based upon that, the agent 1 can predict current values of the agent' 2 state variable as:

$$x_{21} = \chi_2 + \sigma_2 L(t), \qquad < L(t) >= 0, \qquad < L(t)L(t') = 2\delta(t - t') \qquad (9)$$

where L(t) is a Langevin force represented by a random function with zero mean and a δ-correlation function, i.e. the random force has no bias, and its next values are totally independent upon all the previous values. According to this representation the variable x_{12} has the expected value χ_2 and the variance σ_2. In other words, the prediction consists of random guesses dispersed symmetrically around the expected value while the expected value x_2 as well as the variance σ_2 characterizing this dispersion are to be found. Substituting Equation (9) into Equation (8), one arrives at the following Langevin-type stochastic differential equation, (H. Risken, 1989)

$$\dot{x}_1 = a_{11}x_1 + a_{12}(\chi_2 + \sigma_2 L) \qquad (10)$$

of the agent 1. Similar equation can be written for the agent 2

$$\dot{x}_2 = a_{21}(\chi_1 + \sigma_1 L) + a_{22}x_2. \qquad (11)$$

However, from the viewpoint of the agent 1, the last equation may have two different forms

$$\dot{x}_2 = a_{21}x_1 + a_{22}(\chi_2 + \sigma_2 L) \qquad (11a)$$

or

$$\dot{x}_2 = a_{21}(\chi_1 + \sigma_1 L) + a_{22}(\chi_2 + \sigma_2 L) \qquad (11b)$$

Equation (11a) expresses that the agent 1 assumes that the agent 2 knows the state variable of his opponent (or partner), i.e., x_1. On the contrary, equation (11b) expresses that the agent 1 assumes that the agent 2 does not know the opponent's (or partner) variable and predicts it in the same way in which the agent 1 does. From the viewpoint of the reflection levels, the system (10), (11) is on the first level since each agent uses only the image of his opponent while the systems (10), (11a) and (10), (11b) are on the second level since each agent, in addition, uses the image of the image of the opponent (partner) of itself and his opponent (partner). In order to make our point in the simplest way, we will stay with the first level of reflection, i.e., with the system (10), (11).

Formally these equations are not coupled (unlike their original versions (6), (7)). However, as will be shown below, they are coupled indirectly, via the variables $\chi_1,\chi_2,\sigma_1,\sigma_2$. Indeed, since (6) and (7) can be considered as the Langevin-type stochastic differential equations, the evolution of these variables is governed by the corresponding Fokker-Planck equation (H. Risken, 1989)

$$\frac{\partial p}{\partial t}+\frac{\partial}{\partial X_1}[(a_{11}X_1+a_{12}\chi_2)p]+\frac{\partial}{\partial X_2}[(a_{21}\chi_1+a_{22}X_2)p]=$$

$$=(a_{12}^2\sigma_2^2\frac{\partial^2}{\partial X_1^2}+a_{21}^2\sigma_1^2\frac{\partial^2}{\partial X_2^2})p \qquad (12)$$

Here $p(t,X_1,X_2)$ is the joint probability density of distribution of the state variables x_1 and x_2 over the space coordinates X_1 and X_2. Equation (12) describes the evolution of the initial probability density as a result of action of the random forces L(t). This equation must be complemented by the normalization condition

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}pdX_1dX_2=1 \qquad (13)$$

as well as by the definitions of $\chi$and σ

$$\chi_i=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}X_i p(t,X_1,X_2)dX_1dX_2, \qquad i=1,2. \qquad (14)$$

The system (12)-(15) is closed, and it can be solved subject to the initial and boundary conditions

$$\sigma_i=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(X_i-\chi_i)^2 p(t,X_1,X_2)dX_1dX_2, \qquad i=1,2. \qquad (15)$$

$$p(0,X_1,X_2,)=p^0(X_1,X_2), \qquad (16)$$

$$p(t,\pm\infty,\pm\infty)=0, \quad \frac{\partial p}{\partial X_i}(t,\pm\infty,\pm\infty)=0, \qquad i=1,2 \qquad (17)$$

Substituting (14) and (15) (as the known functions of time found from the solution of this system) into equations (10) and (11), one obtains the solutions for the state variables.

$$x_i=\exp(a_{ii}t)[\int_0^t a_{ij}(\chi_j+\sigma_j L)dt+x_i^0], \qquad i,j=1,2; \qquad i\neq j. \qquad (18)$$

Here x_i^0 is the initial value of the corresponding state variable.

It should be noticed that the solutions (18) are random because of the randomness of the Langevin forces L. That is why for qualitative analysis it is more convenient to stay with the statistical invariants of these solutions i.e., with the means χ_i the variances σ_I expressed by Equations (14) and (15), respectively.

It should be noticed that the same strategy of the solution can be applied to the models describing the second level of reflection, i.e., Equations (10), (11a) or (10)(11b) with the only difference that these equations express only the view of the agent 1. After interchanging the indexes 1 and 2 in these equations, one arrives at the similar system expressing the view of the agent 2. Then each agent is supposed to create the image of the model of his opponent (partner), etc.

3. Dialog as evolutionary game with incomplete information.

In this section we will apply the model of two interacting agents presented by equations (10)-(17) to the evolutionary games. The model is intentionally trivialized to make the interaction between the agents easily tractable; at the same time, this model still preserves the distinguished properties of the proposed approach. The general model with many other applications has been discussed in Zak, M, 2003. A game here is understood as a special type of an interaction between two agents, which follow certain rules and expecting a certain outcome. A game is evolutionary if the interacting agents change their internal states in order to be successful in the future. Turning to equations (10) and (11), one can see that this model of interacting agents satisfies both of these definitions. Indeed, the state variable x_I can be associated with internal representations of the agents. The right-hand parts of equations (10) and (11) can be considered as the statements made by each of the agent, respectively. Each of those statements depends upon the both state variables as the reactions to the corresponding statements. The left-hand parts of equations (10) and (11) express the changes of the state variables. From these changes the agents calculate the next statements, and that changes the internal states of the agents. It should be noticed that the representations as well as the statements are arbitrary with respect to what one wants to present. In other words, the model captures the grammar without a semantic. It should be recalled that games could be adversary or cooperative. In this section we will deal only with the cooperative games; in particular, the outcome of the game will be to approach the common ground (or mutual belief, or sheared conception) regardless of the initial conditions. Obviously, if the system (10), (11) is stable, i.e., if

$$a_{11} + a_{22} < 0, \qquad a_{11}a_{22} > a_{12}a_{21}, \tag{19}$$

any initial conditions x_i^0 ($i = 1, 2$) will lead to the common ground i.e., to the zero solution, under the condition that each agent has a complete information not only about the values of his own state variable, but about the values of the state variable of another agent as well. However, in case of language communications, the information is never complete: it could always be interpreted in many different ways, unless the sender and the receiver have some "expected" mutual belief following from previous knowledge about each other backgrounds, or about the context of the forthcoming dialog. Hence, it would be reasonable to assume that although each agent does not know the values of the state variable of his partner, he, nevertheless, can come up with some random guesses that are characterized by known statistical invariants such as the mean χ and the variance σ. Turning to equation (9), one can recognize that this is exactly the same representation we just mentioned. As shown in the last section, actually it is sufficient for the agent to know only the initial values of these invariants since then their current values are uniquely determined by the corresponding Fokker-Planck equation (12). Thus, we have to return to equations (12)-(17) in order to find these invariants. We will demonstrate that for a simple system such as equations (10), (11), one does not need to find the solution to equation (12) subject to the conditions (13)-(17); instead, the direct equations with respect to the statistical invariants χ and σ can be derived from Equation (12). Indeed, let us multiply equation (12) by X_i (i = 1, 2) and integrate over the whole space. Then, taking into account the conditions (13) and (17), one obtains the following system of ordinary differential equations with respect to the expected values of the state variables.

$$\dot{\chi}_1 = a_{11}\chi_1 + a_{12}\chi_2, \tag{20}$$

$$\dot{\chi}_2 = a_{21}\chi_1 + a_{22}\chi_2, \tag{21}$$

Obviously the systems (20), (21) and (6), (7) are identical, i.e., the evolution of the state variables (with complete information) and their expectations is described by the same model. (Such a coincidence results from the linearity of the original mode). But it should be noticed that the stability of expectations does not guarantee the stability of the system (10), (11), i.e., the stability of the state variables with incomplete information. Indeed, let us turn to the solution (18) of equations (10) and (11). Obviously this solution is unstable if $a_{11}>0$ or $a_{22}>0$, regardless of the expectations and variances as functions of time, and even if the first inequality in (19) is satisfied. Therefore, in order to provide the stability of evolution of the state variables with incomplete information, the expectations must be "more stable," i.e., (as it follows from the standard theorems of linear stability theory) the inequalities are to be stronger than (19)

$$a_{11} < 0, \quad a_{22} < 0, \quad a_{11}a_{22} > a_{12}a_{21} \tag{22}$$

In our further analysis we will assume that the conditions (22) hold. However, one should notice that these conditions are only necessary, but not yet sufficient for the stability of equations (10) and (11). In order to derive the sufficient conditions, one has to analyze the evolution of the variances. For that purpose, let us multiply equation (12) by X_i^2 (i =1, 2), and integrate it over the whole space. After transformations similar to those performed above for expectations, one arrives at a system of ODE that is nonlinear with respect to variances and is coupled with equations (20), (21). For better observability, we will simplify this system by assuming that

$$\chi_1{}^0 = 0, \quad \chi_2{}^0 = 0 \tag{23}$$

Then the governing equations for the variances can be written in the following form

$$\dot{\sigma}_1{}^2 = 2a_{11}\sigma_1{}^2 + 2a_{12}{}^2\sigma_1{}^2\sigma_2{}^2, \tag{24}$$

$$\dot{\sigma}_2{}^2 = 2a_{22}\sigma_2{}^2 + 2a_{21}{}^2\sigma_2{}^2\sigma_1{}^2. \tag{25}$$

Although this system is still nonlinear, its stability analysis is simple. Indeed, since it has one attractor

$$\sigma_1 = 0, \quad \sigma_2 = 0, \tag{26}$$

and one repeller

$$\sigma_1^2 = -\frac{a_{22}}{a_{21}^2}, \qquad \sigma_1^2 = -\frac{a_{11}}{a_{12}^2} \tag{27}$$

the stability will be provided if the initial variances are inside the basin of attraction, i.e., if

$$(\sigma_1^0)^2 < -\frac{a_{22}}{a_{21}^2}, \qquad (\sigma_2^0)^2 < -\frac{a_{11}}{a_{12}^2} \tag{28}$$

Thus, the inequalities (22) and (28) are necessary and sufficient for the stability of the system (10), (11) in the simplified case (23). As follows from (28), the incompleteness of the information measured by the initial variances σ_1^0 and σ_2^0 is directly responsible for the divergence of the dialog. At the same time, the stability of the original model (6), (7) represented by the conditions (22), increases the allowed degree of the incompleteness that would still preserve the convergence of the dialog.

In conclusion of this section, we will make several remarks. First we will analyze the effect of noise upon the stability of the dialog. For that purpose, we will add additional Langevin forces to equations (6) and (7):

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + b_1^2\Gamma_1(t), \tag{29}$$

$$\dot{x} = a_{21}x_1 + a_{22}x_2 + b_2^2\Gamma_2(t) \tag{30}$$

Here $\Gamma_1(t)$ and $\Gamma_2(t)$ are random functions with zero means and with correlation functions equal to a δ-function, and the constants b_1^2, b_2^2 represent the strengths of the noise interference for the agent 1 and 2 respectively. After the transformations similar to those performed above, one arrives at the following stability conditions:

$$(\sigma_1^0)^2 < -\frac{a_{22}}{a_{21}^2} + \frac{b_1}{a_{21}^2}^2, \qquad (\sigma_2^0)^2 < -\frac{a_{11}}{a_{12}^2} + \frac{b_2^0}{a_{12}^2}. \tag{31}$$

These conditions demonstrate that the noise interference decreases stability of the dialog by decreasing the upper bound of the initial variances (which are proportional to the degree of incompleteness of information) that are sufficient for the convergence of the dialog to the common ground.

Second, we will assume that each agent is able to stabilize the dialog by applying control (or self-supervising) forces composed of the probability density and its space-derivatives. In order to suppress noise we will choose the following control forces

$$F_1 = -c_1^2 \frac{\partial}{\partial x_1} \ln p, \qquad F_2 = -c_1^2 \frac{\partial}{\partial x_2} \ln p \tag{32}$$

to be added to the right-hand sides of equations (6) and (7) respectively

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + F_1, \tag{33}$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + F_2 \tag{34}$$

Here the constants c_1^2 and c represent the strengths of the control forces. Their contribution into the stability conditions is

$$(\sigma_1^0)^2 < -\frac{a_{22}}{a_{21}^2} + \frac{b_1^2}{a_{21}^2} - \frac{c_1^2}{a_{21}^2}, \qquad (\sigma_2^0) < -\frac{a_{11}}{a_{12}^2} + \frac{b_2^2}{a_{12}^2} - \frac{c_2^2}{a_{12}^2}. \tag{35}$$

As follows from (35), the control forces can completely suppress the effect of noise if

$$c_1 = b_1, \quad c_2 = b_2 \tag{36}$$

Thus, the proposed model consists of three basic components. The first component is the dynamical model of the agents interaction (in the form of a dialog) represented by ODE (see equations (6) and (7)). This model can be associated with the motor dynamics. Since language communications are always incomplete, the motor dynamics has to be complemented by additional information. This information is coming from the collective mind that represents a special knowledge-base (or a context) composed of the abstract images of the agents in terms of their joint probability density; these images capture general characteristics of the agents, their "habits," expected routes of their evolution, possible deviations of the state variables from their expected values, etc., (see equation (12)). From the collective mind (12), each agent can extract the evolution of his own (marginal) probability density p_i.that represents his mental dynamics. In our model, the marginal densities are approximated by the expectations χ_i and the variances σ_i, and therefore, the mental dynamics of the agents is expressed by equations (20), (24), and (21), (25), respectively. The last component of the model is the feedback from the collective mind (12) to the motor dynamics (6), (7) implemented by the control forces (32) (see equations (33) and (34)). These self-supervising forces can be associated with the agent's self-awareness since they are composed only of the internal parameters characterizing the state of the collective mind, while their goal is to effect the motor dynamics in order to achieve a desired out- come of the "game."

The model introduced in this section is, probably, the simplest one that still allows us to present the proof-of-concept using well-observable closed-form analytical solutions. In the next section we will sketch an extension of this model to the general case when only a numerical approach can be applied.

4. General Model

In this section we will generalize the two-dimensional model described by equations (1) and (2) to n-dimension and add the control forces generalizing the forces (32)

$$\dot{x}_{i|i} = f_i(x_{1|i}, x_{2|i}, \dots x_{n|i}) + \sum_{j=1}^{n} \alpha_{ij} \frac{\partial}{\partial x_{j|i}} \ln p(x_{1|i}, \dots x_{n|i}) \tag{37}$$

$$\dot{x}_{j|i} = f_j(x_{1|j|i}, \dots x_{n|j|i}) + \sum_{j=1}^{n} \alpha_{ij} \frac{\partial}{\partial x_{j|i}} \ln p(x_{1|j|i}, \dots x_{n|j|i}), \quad j \neq i \tag{38}$$

Here $x_{k|j|i}$ is the state variable of the image of the k-th agent in view of the j-th agent in view of the i-th agent. This two-reflection-system of n^2 equations is still open since additional equations for the three-index-variables are needed. Such equations can be written in the same form as equations (38), however, they will include the four-index-variables, and the total three-reflection-system will still be open. After m reflections one will arrive into n^{m-1} equations with respect to n^m variables. In order to close the system, one has to introduce a mechanism to create the chain of images. For that purpose, let us turn to the Liouville equation (25) that describes the evolution of the joint probability density $p(X_1, \dots X_n)$. In the same way in which the density $\rho(X)$ in equation (7) describes the self-image of a single agent, equation (25) can be exploited for description of the chain of images in the form of linear regressions

$$x_{j|i} = M(x_j) + \beta_{ji}[x_i - M(x_i)], \quad x_{k|j|i} = M(x_{k|j}) + \beta_{kji}[x_i - M(x_i)], etc \tag{39}$$

Here $M(x_j)$ is expected value of x_j, and the regression coefficients β_{ji}, β_{kji}, etc are uniquely defined by the components of the dispersion matrix $[r_{ij}]$ (for instance, $\beta_{ij} = r_{ij}^2/r_{ii}$). Thus, all the regressions (39) are determined by the distribution ρ governed by equation (25). After substitution the regressions (39) into equations (37) and (38), one arrives at a closed system of ODE's and PDE that couples the motor and mental dynamics in view of the i-th agent.

$$\dot{x}_{i|i} = F_i(x_i, Mx_1, ...Mx_n, r_{ks}) + \frac{\partial}{\partial x_i} \ln \rho_i(x_i, Mx_1, ..Mx_n, r_{ks}) \sum_{j=1}^{n} \alpha_{ij}[1 + \beta_{ij}(r_{ks})], i = 1, ...n. \qquad (40a)$$

$$\dot{x}_{j|i} = F_j(x_i, Mx_1, ...Mx_n, r_{ks}) + \frac{\partial}{\partial x_i} \ln \rho_i(x_i, Mx_1, ...Mx_n, r_{ks}, t) \sum_{j=1}^{n} \alpha_{ij}[1 + \beta_{kji}(r_{sq})], i = 1, ...n. (40b)$$

$$\frac{\partial \rho_i}{\partial t} = -\frac{\partial}{\partial x_i}\{(\rho_i \sum_{j=1}^{n} F_j) + \frac{\partial}{\partial x_i}[\ln \rho_i] \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij}(1+\beta)]\}, \qquad i = 1, ..n, \qquad (41)$$

Here ρ_i is the joint probability density in view of the i-th agent.

It should be emphasized that equation (41) is characterized not only by nonlinear diffusion, but by a nonlinear drift as well since both of these coefficients depend upon the density moments. Obviously, different agents have different views of the system, and these differences start with different initial conditions and initial densities for different i-ths. Hence, all the systems (40)-(41) evolve independently for different i-ths until an external event couples them.

The structure of the complexity chains, in general, can be different from those described above. Indeed, suppose that all the agents, except the i-th one, shear information about the values of their state variables; therefore, they have to predict only the position of the i-th agent. That can be done via linear regression of x_i onto the rest of the variables

$$x_i = M(x_i) + \sum_{k \neq i} \beta_{ik}[x_k - M(x_k)], \; \beta_{ik} = -\frac{\lambda_{ik}}{\lambda_{ii}}, \qquad [\lambda_{ik}] = [\sigma_{ik}]^{-1}. \qquad (42)$$

This structure can be generalized to the case when all the agents are divided into several groups such that the agents in the same group shear all the information about their state variables, while the agents from different groups do not. Thus, the proposed model is capable to capture complexity that matches the complexity of life, and that includes behavior of ecological, social as well as economics systems.

Let us discussed in more details the control (or self-supervised) forces F_i. In our previous model, (see Equations (32)), their role was to stabilize the dialog by suppressing noise. In general, their choice depends upon the objective of the cooperating agents. If this objective is formulated in terms of minimization of a functional

$$J = \int \Phi(p, \nabla p, ...) \, dV \to \min,$$

then, applying the formalism of the control theory, one can find the corresponding control forces. However, one should notice that in the classical control theory, the control forces depend upon the state variables (S. Banks, 1986), while here they are composed of the parameters of the collective mind such as expected values and variances of the state variables, and that is why it is better to call them self-supervised forces.

When communicating agents simulate the human society, some emerging objectives can take over and govern the agent's behavior. As an example, consider the principle of reflectivity introduced by V. Lefebvre (2003): the subject tends to generate a pattern of behavior such that similarity is established and preserved between the subject and his model of the self "while this principle, is a manifestation of a special cognitive mechanism of self-representation rather than a result of the intellectual efforts of the subject consciously thinking about the self." In terms of our formalism, the following self-supervised forces can implement this principle (see M. Zak, 2003)

$$F_i = \gamma_i(\chi_i - x_i)^{\frac{1}{3}}, \qquad \gamma_i = const. \tag{43}$$

On a large time-scale, one can introduce the principle of maximum complexity stating that a human-like community of agents evolves toward the maximum increase of the complexity of its social structure. In terms of our formalism, such an evolution is achieved by the increase of the number of the levels of reflections, (see Equation (5)). However, a natural constraint for such an increase is the exponential growth of the capacity and resources required.

More practical approach to selection of the self-supervised forces can be adopted from the concept of learning in neural nets. Suppose that these forces are sought in the following parametrised form

$$F_i = \tanh\sum_{i,j} w_{i,j} p_j, \qquad i.j = 1,2,\ldots k,$$

in which w_{ij} are constant weights, and p_j is the value of the probability density at a fixed point j of the n-dimensional space $X_1 \ldots X_n$,while k is the number of the points at which the probability density is discretised. Let us assume that our objective is to teach the artificial communicating agents to make correct decisions in response to unexpected changes in external forces or in the objectives. For that purpose, first we have to find an expert whose responses (either rational or intuitive) will be optimal. Then, comparing these responses with the corresponding responses of the model and applying the back-propagation technique, one can find the optimal weights in the self-supervising forces.

5. Interpretation of the model.

In this section we will present and discuss the interpretation of the proposed collective-mind-based model of communicating agents from the viewpoints of mathematics, and physics. We will also propose possible interpretations from viewpoint of biology, psychology, neuro-science, social dynamics and economy, as well as language communications, control theory, and hardware implementation

A. Mathematical Viewpoint

From the mathematical viewpoint the model is represented by the system of the Langevin-type stochastic differential equations (see Equations (10), (11) or Equation (39)) and the corresponding Fokker-Planck equation (see Equation (12) or Equation (40). The connection between these equations is the following: Equation (39) simulates randomness while Equation (40) manipulates by the values of its probability; therefore, if Equation (39) are run independently many times, and statistical analysis of the corresponding solutions is performed, then the calculated probability density will evolve according to Equation (40). However, the major departure from the classical case here is in the coupling between the Langevin and the Fokker-Planks equations. This coupling is implemented by the self-supervising forces F_i as well as by the expectations χ_i and the variances σ_i of the state variables (see Equation (39)). As a result of this coupling, the Fokker-Planck equation becomes nonlinear with respect to the probability density, and that, in turn, leads to new fundamental phenomena in the probability space (M. Zak, 2003). These phenomena include formation of multi-attractor limit sets as well as formation of shock waves, and solitons.

Both phenomena demonstrate a major departure from linear evolution of probability density. The multi-attractor limit sets allow one to introduce an extension of neural nets that can converge to a prescribed type of a stochastic process in the same way in which a regular neural net converges to a prescribed deterministic attractor. An information-based neural net of that type was introduced and analyzed in Zak, M., 2003. The shock waves and solitons decrees the rate of disorder by slowing down or even reverse the diffusion. Therefore, these phenomena can play an impotent role in self-organization of active systems. Another new phenomenon representing a special form of entanglement of stochastic processes is illustrated by Equations (10) and (11). Indeed, formally the stochastic processes described by these equations are independent since Equation (10) does not depend upon x_2 and Equation (11) does not depend upon x_1. However, these processes are coupled via their nvariantsi (see Equations (20) and (21), and that entangles them in a special nonlinear way. It should be noticed that this entanglement implements connection between the agents by mean of the collective mind.

B. Physical Viewpoint.

From the physical viewpoint, the model represents a fundamental departure from both Newtonian and statistical mechanics. Indeed, firstly, in Newtonian mechanics the evolution of the probability density (described by the Fokker-Planck equation) is always linear, and it never affects the underlying motion of the corresponding physical system. Secondly, in Newtonian mechanics the Fokker-Planck equation only registers the evolution of the probability density without affecting the corresponding equations of motion, and there are no principles that would determine additional feedback forces. Both of these conditions are violated in the proposed model: due to the self-supervising forces F_i, Equations (39) and (40) are coupled, and that, in turn, makes Equation (40) nonlinear. The same coupling between the evolution of the probability density and the corresponding motion in physical space may cause a decrease of the entropy i.e. a progressive evolution that is strictly forbidden by the statistical mechanics of isolated systems. Thus, the proposed model is non-compatible with both Newtonian and statistical mechanics. At the same time, it is fully consistent with the theory of differential equations and stochastic processes. The only conclusion following from that is that this model can display some "non-Newtonian" features. Formal similarity of the proposed model and quantum mechanics is discussed in Zak, M. 2002, and 2003.

C. Biological Viewpoint

From the viewpoint of evolutionary biology, the proposed model illuminates the "border line" between living and non-living systems. The starting point of our biologically inspired interpretation is the second law of thermodynamics that states that the entropy of an isolated system can only increase. This law has a clear probabilistic interpretation: increase of entropy corresponds to the passage of the system from less probable to more probable states, while the highest probability of the most disordered state (which is the state with the highest entropy) follows from a simple combinatorial analysis, (I. Prigogine, 1980). However, this statement is correct only if there is no Maxwell' sorting demon, i.e., nobody inside the system is rearranging the probability distributions. But this is precisely what the self-supervising feedback is doing: it takes the probability density p from Equation (40), creates functionals or functions of this density, converts them into a force and applies this force to the equation of motion, (see the last three terms in Equations (39)). As already mentioned above, because of that property of the model, the evolution of the probability density becomes nonlinear, and the entropy may decrease "against the second law of thermodynamics." Obviously the last statement should not be taken literary; indeed, the proposed model captures only those aspects of the living systems that are associated with their behavior, and in particular, with their motor-mental dynamics since they are beyond of the dynamical formalism. Therefore, such physiological processes that are needed for the metabolism are not included into the model. That is why this model is in a formal disagreement with the second law of thermodynamics while the living systems are not. In order to further illustrate the connection between the life-nonlife discrimination and the second law of thermodynamics, consider a small physical particle in a state of random migration due to thermal energy, and compare its diffusion i.e. physical random walk, with a biological random walk performed by a bacterium. The fundamental difference between these two types of motions (that may be indistinguishable in physical space) can be detected in probability space: the probability density evolution of the physical particle is always linear and it has only one attractor: a stationary stochastic process where the motion is trapped. On the contrary, a typical probability density evolution of a biological particle is nonlinear: it can have many different attractors, but eventually each attractor can be departed from without any "help" from outside.

That is how H. Berg, 1983, describes the random walk of an E. coli bacterium: "If a cell can diffuse this well by working at the limit imposed by rotational Brownian movement, why does it bother to tumble? The answer is that the tumble provides the cell with a mechanism for biasing its random walk. When it swims in a spatial gradient of a chemical attractant or repellent and it happens to run in a favorable direction, the probability of tumbling is reduced. As a result, favorable runs are extended, and the cell diffuses with drift". Berg argues that the cell analyzes its sensory cue and generates the bias *internally*, by changing the way in which it rotates its flagella. This description demonstrates that actually a bacterium interacts with the medium, i.e., it is not isolated, and that reconciles its behavior with the second law of thermodynamics. However, since these interactions are beyond the dynamical world, they are incorporated into the proposed model via the self-supervised forces that result from the interactions of a biological particle with "itself," and that formally "violates" the second law of thermodynamics. Thus, the proposed model offers a unified description of the progressive evolution of living systems. Based upon this model, one can formulate and implement (via the reflective chains) the principle of maximum increase of complexity that governs the large-time-scale evolution of living systems. It should be noticed that at this stage, our interpretation is based upon logical extension of the proposed mathematical formalism, and is not yet corroborated by experiments.

D. Psychological viewpoint.

From the viewpoint of psychology_the proposed model can be interpreted as representing interactions of the agent with the self-image and the images of other agents via the mechanisms of self-awareness. In order to associate these basic concepts of psychology with our mathematical formalism, we have to recall that living systems can be studied in many different spaces such as physical (or geographical) space as well as abstract (or conceptual) spaces. The latter category includes, for instance, social class space, sociometric space, social distance space, semantic space e.t.c.Turning to our model, one can identify two spaces: the physical space x,t in which the agent state variables $x_i(t)$ evolve,(see Equation (39)), and an abstract space $p(X_1..X_n, t)$ in which the probability density of the agent' state variables evolve (see Equation (40)).The connection with these spaces have been already described earlier: if Equation (39) are run many times starting with randomly chosen initial conditions, as well as with random values of the Langevin forces $L_i(t)$, one will arrive at an ensemble of different random solutions, while Equation (40) will show what is the probability for each of these solutions to appear. Thus, Equation (40) describes the general picture of evolution of the communicating agents that does not depend upon particular initial conditions. Therefore, the solution to this equation can be interpreted as the evolution of the self- and non-self images of the agents that jointly constitutes the collective mind in the probability space. Based upon that, one can propose the following interpretation of the model of communicating agents: considering the agents as intelligent subjects, one can identify Equation (39) as a model simulating their motor dynamics, i.e. actual motions in physical space, while Equation (40) as the collective mind composed of mental dynamics of the agents. Such an interpretation is evoked by the concept of reflection in psychology, (V. Lefebvre, 1997). Reflection is traditionally understood as the human ability to take the position of an observer in relation to one's own thoughts. In other words, the reflection is the self-awareness via the interaction with the image of the self. Hence, in terms of the phenomenological formalism proposed above, a non-living system may possess the self-image, but it is not equipped with the self-awareness, and therefore, this self-image is not in use. On the contrary, in living systems the self-awareness is represented by the self-supervising forces which send information from the self-image to the motor dynamics. Due to this property that is well-pronounced in the proposed model, an intelligent agent can run its mental dynamics ahead of real time, (since the mental dynamics is fully deterministic, and it does not depend explicitly upon the motor dynamics) and thereby, it can predict future expected values of its state variables; then, by interacting with the self-image via the supervising forces, it can change the expectations if they are not consistent with the objective. Such a self-supervised dynamics provides a major advantage for the corresponding intelligent agents, and especially, for biological species: due to the ability to predict future, they are better equipped for dealing with uncertainties, and that improves their survivability. It should be emphasized that the proposed model, strictly speaking, does not discriminate living systems of different kind in a sense that all of them are characterized by a self-awareness-based feedback from mental to motor dynamics. However, in primitive living systems (such as bacteria or viruses) the self-awareness is reduced to the simplest form that is the self-nonself discrimination; in other words, the difference between the living systems is represented by the level of complexity of that feedback.

E. Neuro-Science Viewpoint

From the viewpoint of neuro-science the proposed model represents a special type of neural net. Indeed, reinterpreting an agent's state variable x_i as a neuron's mean soma potential, and assuming that each neuron receives full information from the rest of neurons, one arrives at a conventional neural net (37). It should be recalled that in this case the self-supervising forces are not needed, and they can be ignored. The departure from the conventional case starts with the incompleteness of information, i.e., when a neuron does not receive the values of the mean soma potentials from the rest of the neurons. This incompleteness is compensated by a "general knowledge" stored in the collective mind (40) and delivered to the neural net via the self-supervising forces F_i. As a result of that, the neural net (39) becomes random, while the evolution of its statistical invariants is described by the collective mind (49). In order to illuminate the difference between these two cases we will start with a single continuously updated linear neuron with a dissipation feedback

$$\dot{x} = -x, \tag{44}$$

The state variable x eventually approach an attractor x=0 regardless of initial conditions.

$$x = x_0 \exp(-t), \tag{45}$$

In general case, a multi-dimensional nonlinear neural net may converge to one of the several attractors that can be static, periodic, or chaotic as well. However, one fundamental property remains the same: as soon as these attractors are approached, the evolution stops.

Let us turn now to a stochastic extension of a neuron (44). This can be done in several ways. One way is to relax the Lipschitz conditions, (M. Zak, 1992). Another way is to introduce a special types of the equilibrium points which are attractors in one direction and repellers in the others. In the both cases the neuron state variable will perform a Brownian motion that can be included in Equation (44) via the Langevin force L(t):

$$\dot{x} = -x + L(t). \tag{46}$$

Equation (46) has the solution:

$$x = x_0 \exp(-t) + \int_0^t \exp[-(t - t')] L(t') dt'. \tag{47}$$

Equation (47) describes a stochastic process that characterizes the evolution of the neuron state variable. The evolution of the probability density is described by the corresponding Fokker-Planck equation

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial X}(Xp) + D\frac{\partial^2}{\partial X^2}p. \qquad (48)$$

Its solution for the sharp initial value p(x,0)=δ(x→0) is represented by a normal distribution

$$p = \frac{1}{\sqrt{2\pi D'}}\exp[-\frac{X^2}{2D'}], \qquad D' = D[1-\exp(-2t)]. \qquad (49)$$

As t→∞, the distribution tends to the thermodynamical limit with D'→D. Obviously, D>D', and therefore, the entropy E=LnD√2π approaches its maximum value at t→∞. This result can be extended to the general case of multi-dimentional diffusion-based neural nets. That means that the evolution of such neural nets is always regressive, i.e. their entropy can only increase.

Let us introduce a control force to reverse the increase of the entropy. Within the framework of the Newtonian formalism, the most general control force must depend upon the state variables and time, i.e., F = F(x,t). Substituting this force into Equation (46), and introducing the corresponding changes into Equation (48), in which the drift and the diffusion coefficients are functions of X and t. Then, according to the Boltzman H-theorem, (H. Risken, 1989), the entropy of the system will still monotonously increase regardless of the a particular form of the control force. However, the situation is changed if the control is represented by a self-supervised force composed of the probability density and its derivatives. For the proof of concept, let us choose this force as following

$$F = D\frac{\partial}{\partial x}\ln p \qquad (50)$$

which is applied after t>T. Then Equations (46) and (48) are to be rewritten as

$$\dot{x} = -x + L(t) + D\frac{\partial}{\partial x}\ln p \qquad (51)$$

and

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial X}[(X - D\frac{\partial}{\partial X}\ln p)p] + D\frac{\partial^2}{\partial X^2}p, \qquad (52)$$

respectively. After trivial transformations, Equation (52) is reduced to the form in which the diffusion term is suppressed

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial X}(Xp). \qquad (53)$$

The solution to this equation that starts from t>T is

$$p = \sqrt{\frac{\beta}{2\pi Y}} \exp[\ \beta t - \frac{\beta X^2 \exp(\ 2\beta t)}{2DY}, \qquad Y = 1 - \exp(\ -2\beta t). \qquad (54)$$

As follows from this solution, the self-supervised force reverses the evolution of the probability density towards the decrease of the entropy, and that makes a self-supervised neuron a "messenger of life."

Thus, the fundamental property of the self-supervised neuron is its ability to create the self- image (Equation (52)) and interact with this image (Equations (53) and (54)). It would be a challenge for a future research to associate the self-supervised neuron with the mirror neuron 9 recently discovered in the monkey) that fires both when performing an action and when the monkey is observing the same action performed by another subject. Indeed, the way in which the self-supervised neuron works is the following. It is assumed that all the communicating agents belong to the same class in a sense that they share the same general properties and habits. It means that although each agent may not know the exact positions of the rest of the agents, he, nevertheless, knows at least such characteristics as their initial positions (to accuracy of initial joint probability density, or, at least, initial expected positions and initial variances). This preliminary experience allows him to reconstruct the evolution of expected positions of the rest of the agents using the collective mind as a knowledge base. *Hence, a self-supervised neuron representing an agent A can be activated by an expected action of an agent B which may not be in a direct contact with the agent A at all, and that can be associated with the mirror properties of the self-supervised neuron.*

The collective properties of self-supervised neurons, i.e., the self-supervised neural nets have a significant advantage over the regular neural nets: they possess a fundamentally new type of attractor –the stochastic attractor that is a very powerful generalization tool. Indeed, it includes a much broader class of motions than static or periodic attractors. In other words, it provides the highest level of abstraction. In addition to that, a stochastic attractor represents the most complex patterns of behavior if the self-supervised net describes a set of interacting agents. Indeed, consider a swarm of insects approaching some attracting pattern. If this pattern is represented by a static or periodic attractor, the motion of the swarm is locked up in a rigid pattern of behavior that may decrease its survivability. On the contrary, if that pattern is represented by a stochastic attractor, the swarm still has a lot of freedom, and only the statistic of the swarm motion is locked up in a certain pattern of behavior. For example, an information-based neural net (M. Zak, 2003) can approach a stochastic attractor that preserves a prescribed amount of information express via the entropy E.

It should be emphasized that, due to the multi-attractor structure, the proposed model provides the following property: if the system starts from different initial conditions, it may be trapped in a different stochastic pattern. Such a property, in principle, cannot be provided by regular neural nets or cellular automata since they can have only one *stochastic* attractor.

F. Social and Economic Viewpoint

One of the basic problem of social theory is to understand "how, with the richness of language and the diversity of artifacts, people can create a dazzlingly rich variety of new yet relatively stable social structures," (M. Arbib, 1986). Within the framework of the dynamical formalism, the proposed model provides some explanations to this puzzle. Indeed, social events are driven by two factors: the individual objectives and social constraints. The first factor is captured by the motor dynamics (39), while the social constraint is created by the collective mind (40). A balance between these factors (expressed by stochastic attractors) leads to stable social structures, while a misbalance (expressed by stochastic repellers) causes sharp transitions from one social structure to another (revolutions) or to wandering between different repellers (chaos, anarchy). For an artificial "society" of communicating agents, one can assign individual objectives for each agent as well as the collective constrains imposed upon them and study the corresponding social events by analyzing the governing equations (39) and (40). However, the same strategy is too naïve to be applied to a human society. Indeed, most human as members of a society, do not have rational objectives: they are driven by emotions, inflated ambitions, envy, distorted self- and nonself images, etc. At least some of these concepts can be formalized and incorporated into the model. For instance, one can consider emotions to be proportional to the differences between the state variables and their expectations

$$E_m = c(\chi - x). \tag{55}$$

Equation (55) easily discriminates positive and negative emotions. Many associated concepts (anger, depression, happiness, indifference, aggressiveness. and ambitions) can be derived from this definition (possibly, in combination with distorted self and non-self images). But the most accurate characteristic of the human nature was captured by cellular automata where each agent copies the behaviors of his closest neighbors (which in turn, copy their neighbors, etc.). As a result, the whole "society" spontaneously moves toward an unknown emerging "objective." Although this global objective is uniquely defined by a local operator that determines how an agent processes the data coming from his neighbors, there is not known any explicit connection between this local operator and the corresponding global objective: only actual numerical runs can detect such a connection. Notwithstanding the ingenuity of his model, one can see its major limitation: the model is not equipped with a collective mind (or by any other type of a knowledge base), and therefore, its usefulness is significantly diminished in case of incompleteness of information. At the same time, our model can be easily transformed into a cellular automata with the collective mind. In order to do that one has to turn to Equation (37), replace the sigmoid function by a local operator, and the time derivative-by the time difference. Then the corresponding Fokker-Planck equation (40) reduces to its discrete version that is Markov chains, (M. Zak, 2000). On the conceptual level, the model remains the same as discussed in the previous sections. This illustrates a possible approach to the social dynamics based upon the proposed model.

From the viewpoint of economics, the proposed model can represent games with incomplete information. Probably, the best illustration of that is the so called minority game which is a simplified model of conflicting situations observed in financial markets, (D. Challet, 1997). It describes a system in which an odd number N of agents is allowed to make two possible choices: 1 or 0, and that divides the agents in two groups while the group with less number of agents wins. Clearly when agents know nothing about the possible strategies of their adversaries, the outcome is totally random. However, if the agents shear some global information about each other (such as the history of the game in the form of the sequence of the last winning choices), the dynamics of the game becomes extremely complex (for instance, it includes such phenomena as the phase transitions). Within the framework of our model, the sheared information can be stored in the collective mind, and this will provide the agents with the dynamics of interaction between the sheared knowledge and the individual strategies. An exciting challenge for future work is to test the utility of this approach in formalizing shared information as used in section 3.

G. Language communications viewpoint.

Language represents the best example of a communication tool with incomplete information since any message, in general, can be interpreted in many different ways depending upon the context i.e. upon the global information sheared by the sender and the receiver. Therefore, the proposed model is supposed to be relevant for some language-oriented interpretations. Indeed, turning to Equation (39), one can associate the weighted sum of the state variables with the individual interpretations of the collective message made by the agents. The sigmoid functions of these sums form the individual responses of the agents to this message. These responses are completed by the self-supervising forces that compensate the lack of information in the message by exploiting the global sheared information stored in the collective mind, (see Equation (40)). The agent's responses converted into the new values of their state variables are transformed into the next message using the same rules, etc. These rules determined by the structure of Equations (39) and (40) can be associated with the grammar of the underlying language. In particular, they are responsible for the convergence to- or the divergence from the expected objective. It should be noticed that the language structure of the proposed model is invariant with respect to semantics. Hence, in terms of the linguistics terminology that considers three universal structural levels: sound, meaning and grammatical arrangement, (M. Yaguello, 1998), we are dealing here with the last one. To our opinion, the independence of the proposed model upon the semantics is an advantage rather than a limitation: it allows one to study invariant properties of the language evolution in the same way in which the Shannon information (that represents rather an information capacity) allows one to study the evolution of information regardless of a particular meaning of the transmitted messages.

Let us now try to predict the evolution of language communications based upon the proposed model. As mentioned earlier, the evolution of the living systems is always directed toward the increase of their complexity. In a human society, such a progressive evolution is effectively implemented by increase or the number of reflections in a chain "What do you think I think you think, etc." The society may be stratified into several levels or "clubs" so that inside each club the people will shear more and more global information. This means that the language communications between the members of the same club will be characterized by the increased capacity of the collective mind (see Equation (40)), and decreased information transmitted by the messages (see Equation (39)). In the theoretical limit, these messages will degenerate into a string of symbols, which can be easily decoded by the enormously large collective mind The language communications across the stratified levels will evolve in a different way: as long as the different clubs are drifting apart, the collective mind capacity will be decreasing while the messages will become longer and longer. However, the process of diffusion between these two streams (not included in our model) is very likely.

H. Control theory viewpoint.

The proposed model can be considered as a closed-loop controlled dynamical system known in the engineering control, with the only difference that, unlike the engineering control where the control forces are triggered by the values of the state variables and their time-derivatives, here the control forces are determined by the parameters of the collective mind that implicitly represent the state variables of the underlying system. This type of control can be linked to a so-called reflective control introduced in mathematical psychology by V. Lefebvre, 2001, since the system is governed by the reflections, i.e., by the parameters characterizing the images rather than real objects. The mathematical consequences of this property have been discussed in the sub-section A of this section.

I. Applications and implementations.

The proposed model has two types of applications that can be associated with science and technology, respectively. The first type includes theoretical studies of behavior of living systems, and can be performed by direct computer simulations of the system (39), (40). The second type includes the development of artificial living systems that are supposed to simulate and replace some functions of a human (robots, unmanned spacecrafts, etc.). The most effective way of implementation of these systems is by means of analog devices such as VLSI chips used for neural net's analog simulations (C. Mead, 1989). As discussed in the sub-section E of this section, Equation (39) can be treated as regular continuously updated neural nets with additional random forces. Implementation of these forces has been proposed by M. Zak, 2003, based upon non-Lipschitz dynamics. Equation (40), after approximation of space-derivatives be finite differences, can be easily transformed to a neural-net-like dynamical system that can be implemented by VLSI chips.

6. Conclusion.

In summary, we have introduced a new mathematical formalism that offers a rich framework for developing models capturing non-Newtonian properties of living systems. The proposed general approach has been focused on the behavior of communicating agents who compensate an incompleteness of exchanged information by means of the collective mind as a context-type of the global sheared knowledge base. Detailed analyses of an example illustrating the proposed formalism as well as discussion of speculations about its scientific and technological applications have been performed.

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